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ABSTRACT

Arithmetic programming for students with mild mental disabilities requires a comprehensive perspective that includes attention to curriculum, instruction, and appraisal. Arithmetic computation should not dominate educational programming, but should be included in ways that are functionally relevant and meaningfully presented within a framework of problem-solving and applied experiences. The teaching of counting depends on an understanding of two basic concepts, one-to-one and many-to-one relationships. Instruction in these areas can take place in the forms of arrangements, patterns, and sets. A scheme referred to as the Interactive Unit makes instruction both systematic and flexible by organizing instruction, with four instructor inputs (manipulate, display, say, write) and four learner behavior changes (manipulate, identify, say, write). Within the schemata of the Interactive Unit, the teacher can develop an unlimited number of instructional activities and accompanying materials. In the use of arithmetic algorithms, it is important that the teacher become familiar with a variety of algorithms and correct only errors rather than the student's performance style. Other recommended practices include presenting addition as a process of joining sets, and using expanded notation to numerically represent each place value position. Examples are provided to illustrate how these methods can be used in the classroom. Eight figures are included. (JDD)

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Chapter Seven

Arithmetic

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OVERVIEW

This chapter will deal with arithmetic programming for children who have been determined to be significantly below average in mental ability. We will present information of use to teachers of such children if they are willing to adapt existing programs to accommodate the children. Traditional math programming as it exists in regular classrooms has been trimmed, compacted, and watered down for those students who have been classified as Mildly Mentally Retarded. In order to provide these students with the best and most appropriate education in mathematics, teachers must break out of the traditional mold from which they themselves were educated in math, and be willing to be innovative and creative. Mathematical programming for children with mild mental disabilities requires a comprehensive perspective that minimally must include attention to matters of curriculum, instruction and appraisal. This short chapter cannot provide an all-inclusive look at arithmetic programming for mildly handicapped children, but will provide one perspective that is needed to adapt teaching methods to meet their needs.

BASIC CONSIDERATIONS

Curriculum is an essential programming consideration because it determines what will be taught, the sequence and level at which it will be taught, the amount of time that will be devoted to each topic and the contextual scheme (i.e., use of a ditto or a real experience such as shopping) in which the topic will be presented.

Curriculum decisions are paramount in programs for children with mild mental disabilities because known levels of attainment (Cawley & Miller, in progress; Cawley, Miller and Watts, in progress; Cawley & Goodman, 1968) indicate only modest levels of acquisition and facility in applying skills to problem solving (Grise, 1980) and to real-life situations (Levine & Langress, 1985). Therefore, the curriculum decision, or what, when and where certain topics shall be covered, is fundamentally our most important issue. We need to choose those components of mathematics that meet more than the traditional single criterion that there are sufficient dittos available to keep the students busy. What we need to do is to select and teach those elements of mathematics that meet the needs of individuals as children and as adults. If we were truly to do so, we would likely eliminate dittos that stress rote computation from a particular school program.

Instructional considerations must be capable of by-passing the effects of one disability or obstacle, and they must also provide a systematic and flexible system for teacher presentation and learner performance across manipulative, pictorial, spoken, and written alternatives. Ironically, it is the latter type that dominates school programming while the initial three dominate mathematics external to school.

Appraisal enters into our program equation in an important manner. It is appraisal that will provide curriculum specialists and instructional personnel with a representation of the child to guide provision of an appropriate education. This will not be provided through single topic standardized tests such as the Wide Range Achievement Test (Jastak & Jastak, 1978) that measures computation, or through Curriculum-Based Assessment approaches (e.g., Marston & Magnusson, 1985) that do the same unless, of course, computation is our only interest.

BEST PRACTICES

While arithmetic computation is the basis for the preparation of this chapter, we do not agree that programs for children with mental disabilities should be dominated by computation. We do however, propose two appropriate functions for computation in the curriculum for the mentally disabled child. We take the position that computation is valid for school purposes in part due to its value in determining the correct answer in problem solving. The second reason computation is valid for school purposes is its assistance to the individual in estimating or determining an acceptable response in real-life situations.

We reject the proposition that computation is valid because children need to complete reams of dittos or that they have to pass certain tests of basic skills. The Florida data (Grise, 1980) and that of the Everyday Cognition specialists (Levine & Langress, 1985) clearly show that what we do by way of computation in school is of little or no long-term value to children with mental disabilities. Recognizing this, the State of Connecticut (Carter & Leinwand, 1987) has gone so far as to provide 35,000 hand-held calculators to nonhandicapped and handicapped children when they take the state competency test. This allows for a concentration on problem-solving and the role of computation is subordinated.

What we support is functionally-relevant computation, conceptually and meaningfully presented within a framework of problem-solving and applied experiences. Two brief vignettes support our proposal.

1. During the 1986-1987 academic year we (Cawley & Miller, in progress) conducted a demonstration project in science and mathematics for elementary school handicapped children. We employed a regular science and a regular math teacher, neither of whom had any special education background. Each day, Ms. Sandra Bennett, the mathematics teacher taught two self-contained class periods and four resource periods. These segments included children with mental disabilities, behavioral difficulties and a myriad of learning disabilities. Throughout the school year this teacher did not use a single "ditto master" of the traditional computational type. In one 9 week period of work with primary age children with mental disabilities - defined in Louisiana as two standard deviations from the mean - this teacher approached computation from a problem-solving and conceptual perspective every day. The dominant medium was animal crackers and the children learned to compare, to match, to make greater or fewer by certain criteria, to join and to separate and to tabulate. The regular special education teacher could not believe the complexity of the interactions the children were displaying with the basic operations. She kept waiting for the dittos, when, in fact, none were needed.

2. Our second vignette involves six secondary school age youngsters with mild mental disabilities according to the Louisiana criteria described above. Each of these youngsters was given a 92 item test of arithmetic computation. Success was limited to items that did not require renaming. When asked what each would like to learn, they all responded with "division." Their teacher, Ms. Diane Noto, taught them meaningful division in a way that excluded subtraction and multiplication. Once they demonstrated a complete understanding of division, they were

introduced to calculators to minimize the computational burden. As follow-up, the children and the teacher spent the entire 1986-1987 school year working on units that focused on applications and solutions to social problems. Computers and calculators replaced the ditto.

Our two vignettes indicate that it is possible to provide long term and systematic instruction without the ever present workbook or ditto complete with computation items. They also indicate that it is possible to teach children within contextual settings and to focus on meanings and understandings in place of rote learning. They also indicate that it is possible to teach any of the four basic operations on whole numbers in different sequences and that our absolute dependency on addition to subtraction to undertake multiplication to division is unwarranted. Finally, they indicate that it is possible for children to learn about computation in a manner that clears the way for extended nonpaper-pencil activities and the eventual use of calculators.

Counting

Counting by one or more to identify the cardinal property of a set and counting to ten in its many combinations (e.g. ten 10's equal 100) are two skills necessary for effective computation. The former provides the basis for single digit operations and the latter provides the basis for combinations of two or more digits and renaming.

Basic Concepts

Two basic concepts, one-to-one and many-to-one relationships form the basis for counting. Instruction in these areas can take place in the forms of arrangements, patterns and sets.

Arrangements (see Figure 1) provide experiences in one-to-one relations via perceptual and spatial configurations. Making an object as long or the same as one shown (item A) when given the same number of blocks or other objects does not invoke counting, per se. Individuals can make the match by simply aligning the objects with one another. Later, different numbers can be given to the children (item B) and they can observe relative to the extent to which they provide only the correct match. Item C shows two triangles of different sizes. The children could be given sticks of a length equal to the size of a side in the small triangle and instructed to place the sticks on the lines so they make the triangle (this is also a good experience for congruence). They could then be shown the larger triangle and instructed to put the sticks on the line so they make that triangle. Here two smaller sticks would be required and the child would be working with many-to-one relations.

Additional one-to-one activities can be done with item D, only this time items of different length would be utilized.

Patterns are somewhat different from arrangements in that there are specific rules that guide the construction, matching or symbolization that takes place. Figure 2 shows two illustrations. In the first illustration (A), the one-to-one correspondence is developed by matching alternating combinations (i.e., a black box/a white box) and the child must build the figure by placing black/white or white/black combinations. The second illustration (B), requires the child to build the alternating combination but

this time the items are all of the same color, but different shapes. Thus, the child must build the triangle/circle or circle/triangle combination. Most of you as readers, will note that the task in the second illustration is what Zeaman and House (1963) described as an interdimensional shift. Shifts were integral facets of their research on attention theory among children with mental disabilities. We thus see that it is possible to integrate the findings and methods of some of our most basic research into our curriculum and instructional programming.

Sets used with or without specific arrangements offer an excellent opportunity to begin the actual counting process and to provide numerous instances for generalization. Sets are just about anything anyone wants them to be. We focus on sets with one-to-one correspondence and sets with many-to-one correspondences. The instructor can put down a number of objects and the child can be requested to put down as many. The objects can be of any category or type. We see this in two basketball teams with 5 players each or one carton of milk for each child at the lunch table. Many-to-one correspondences are beautifully represented with sets. We see the many apples on a single tree, the four tires on an automobile, the two tires on a bike, the large number of cars on a single street, which we often call a traffic jam, an even larger number of people in a single football stadium and an even larger number of grains of rice in a jar. Each member of a group can be given a jar and each can be requested to put the same amount of rice in his/her jar. They can do this by the spoonful or handful and make many-to-one correspondences perceptually. What we need them to understand is that there are numerous grains in each jar and that they agree on their sameness on a perceptual basis. One could always ask them to count, but in such a case one should use very small jars.

One-to-one and many-to-one correspondences are all about us, but the children fail to attend to these. Show them an automobile with only three wheels and they will tell you that it won't run. Intuitively, they know these forms of relationships. What we need to do is highlight them in mathematical terms and to heighten the attention of the children to them.

Counting

Kids like to count. They also like to say the numbers in sequence. Our job is to bring the two into their proper alignment so that when the children say "five" they mean that 5 items represent the "howmanyness" of a set and they mean this regardless of the types of objects in the sets.

We recommend the following stages for use in beginning counting.

1. Counting to name a set after it has been formed.

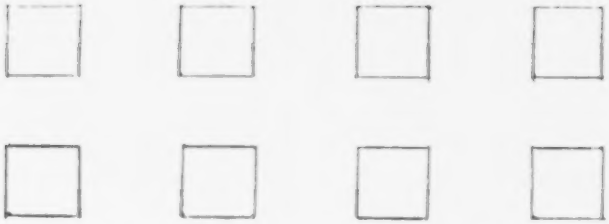
   The teacher takes some objects and places them on the table and says, "I have three. See my three _____ here. Tell me how many I have here." The child says, "Three."

2. Counting to produce a set.

   The teacher places a set of objects on the table and says, "Show me three _____." The child counts out three objects and the teacher says, "Good, there are three _____".

Figure 1

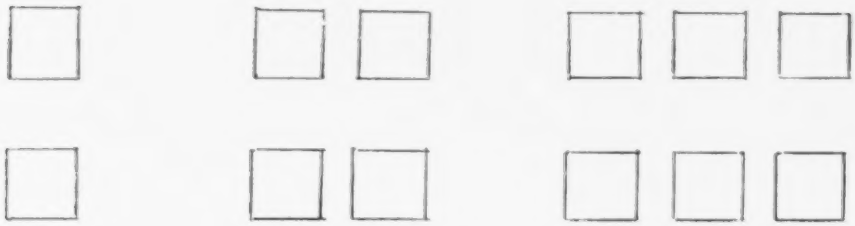
A.



Example

Child's Response

B.



Example

Child's Response

C.

lay sticks (3) over

lay sticks (6) over (2 per side)

D.

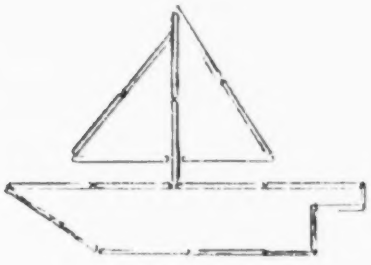
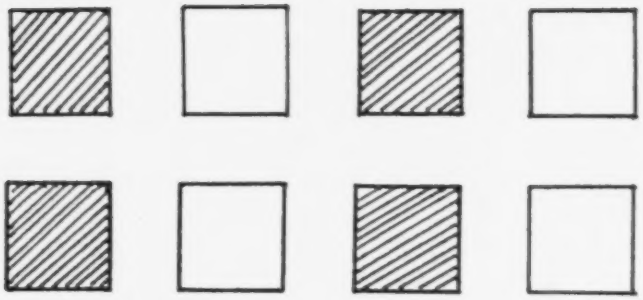


Figure 2

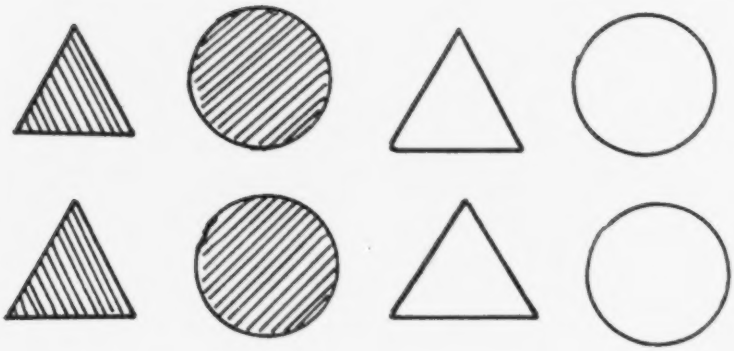
A.



Example

Response

B.



Example

Response

3. Counting to identify a set with that many.

   The teacher places two or more sets of objects on the table and says, "See these. Point to the one that has three." Child points.

4. Counting to name the set.

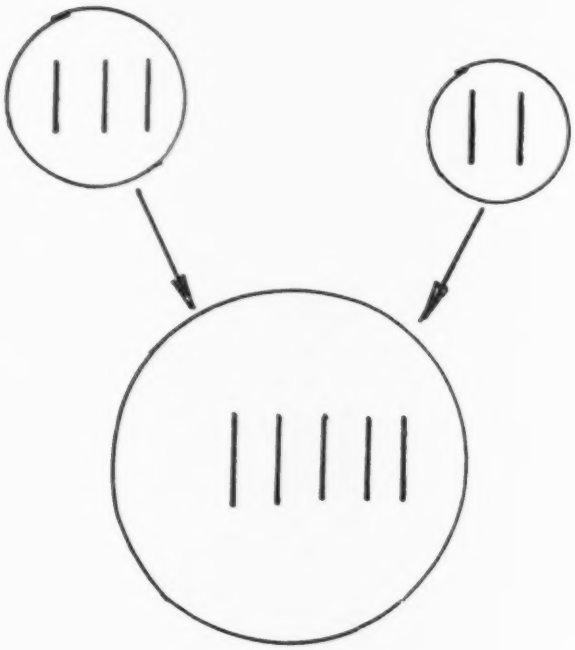
   The teacher places some objects on the table and says, "See these. How many do you see?"

There are two additional stages we deem important for counting. The first is "counting on" to join and to name two or more sets as one. The teacher places two or more sets of objects on the table as shown in Figure 3A. The teacher asks the child to count the number of items in each set separately. The teacher then pushes the two sets together and asks the child to count the number of items in the new set.

The teacher next places two or more sets on the table and asks the child to tell how many there are altogether, first by counting the number in one set and then by counting on to the next set, Figure 3. The child counts the first set and says, "Three." As soon as this is done, the teacher removes the set and asks the child to go on to the next set where he counts (four/five) and says, "Five." Our goal is to get the child to "count on." We want to inhibit the type of counting where the child counts the objects in the first set, then counts the objects in the next set and then begins again with the first set to count all the way through. The initial set can be held in the teachers hand and removed as soon as it is counted. Objects can be placed in paper bags or even hidden in different places in the room. The child counts the number in one location, goes to the other and tells how many altogether.

Figure 3

A.



Count each set.

Count items

B.

Child counts on--
i.e. 1,2,3
then 4,5.

The second important element is the counting of "ten." Activities at this stage begin with the counting of sets with ten or more objects. Each time ten is reached the child is to set this many aside or wrap them with a rubber band. Counting to ten initiates the move to renaming and to place value. Children need to understand the interrelationships between ten 1's as one 10 and one 10 as ten 1's. The children described in our second vignette were able to describe ten 1's as one ten when they counted each stick individually and wrapped them with a band. However, when the band was removed and they were asked how many they had, they found it necessary to count the sticks. When students fully understand this relationship, they are ready to join two or more sets whose sum is greater than 9.

As soon as the children are able to count ten and understand the interrelationships among ten 1's and one 10, they should begin counting by ten and eventually understand the interrelationships among ten 1's and one 10 and ten 10's and100 and one hundred 1's as ten 10's. Once this level has been reached, they are ready to join two or more sets which have sums greater than 19, but less than 100. They need to know that 46 is the same as 30 plus 16, that 23 plus 46 is the same as 20 + 3 plus 40 + 6. Children will understand these relationships naturally if the instruction takes place with set of counting objects (e.g. toothpicks). They will not see the relationships if the instructional approach is dominated by paper-pencil activities.

Systematizing Instruction

For many years, our approach to instruction has been guided by the principle that one must be both systematic and flexible. In the classroom or resource setting, we have used a scheme referred to as the Interactive Unit to operationalize our guiding principle. Figure 4 shows the Interactive Unit (Cawley, 1985).

As can be seen there are 16 combinations of teacher/learner interactions with an innumerable number of activities that can take place within each.

The Interactive Unit (Cawley, 1985) meets needs by:

1. systematically varying input/output relations

2. partialling out the effects of one difficulty (e.g. reading) on performance in another area (e.g. math)

3. providing equivalence among manipulative and symbolic representations to develop conceptual learnings

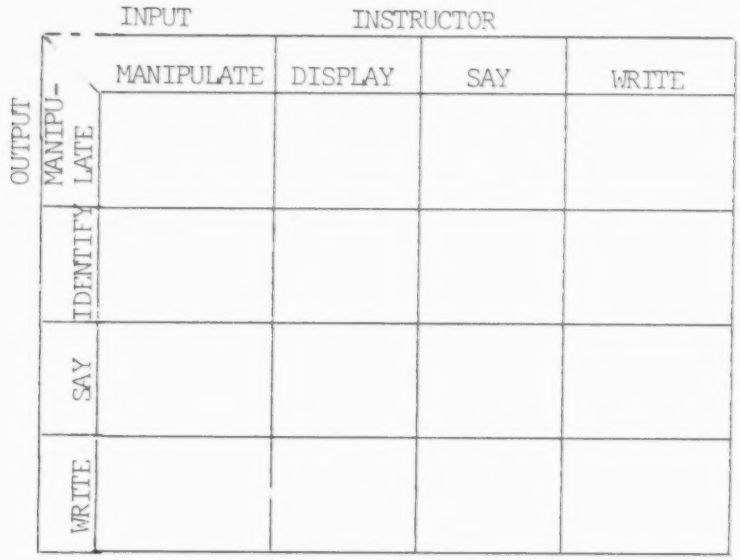
4. guiding the development of an unlimited amount of instructional tasks and materials

The IU consists of rows and columns. As one goes across a row, it is the teacher behavior or input that changes. As one goes down a column it is the learner or response behavior that changes. The teacher can select any input or response combination to meet the needs of the children and to develop the meanings that underlie the operations.

Figure 4

Interactive Unit

Manipulate .... Manipulation of objects (piling, arranging and movement)
Display ..... (Instructor Interaction Presentation of displays (pictures, arrangement of materials)
Say..... Oral discussion
Write ..... Written materials (letters, numerals, words, signs of operation) and marking of these types of materials
Identify..... Learner Interaction Selection from multiple choices of nonwritten materials (pictures, objects)



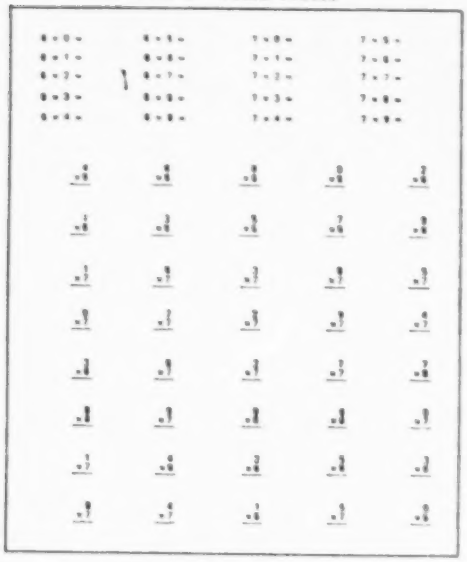
The IU is not hierarchical. Manipulation is not viewed as a lower level of cognition or understanding than symbolic representations. To illustrate, we would wager that more children can do the long division algorithm via paper-pencil than there are those who can do it with toothpicks. Further, there are more errors and deviant representations with paper-pencil than there are with manipulatives. The reason for this is likely our misguided notion of "concrete." Special educators have for years been told that to be concrete is synonymous with the manipulation or the use of objects. Objections to such an interpretation were raised many years ago (Cawley & Vitello, 1972) and they remain today.

Manipulatives are frequently used to teach in a rote manner. Clearly, rote use of manipulatives or the rote learning of basic facts and paper-pencil algorithms (e.g., start with the column on the right; Pelligrino & Goldman 1987) are more intellectually damaging to the handicapped child than a meaningful representation with blocks.

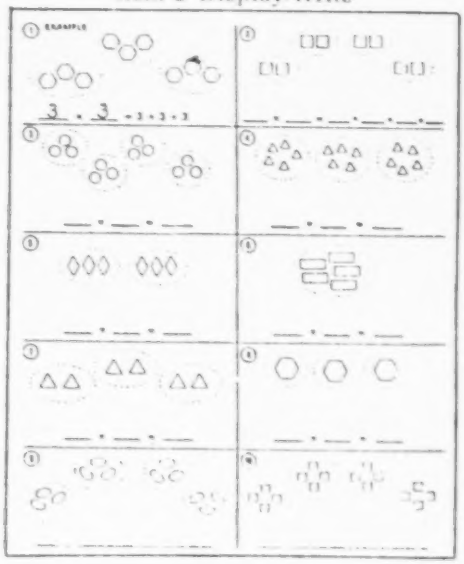
Within the schemata of the IU, the teacher can develop an unlimited number of instructional activities and accompanying materials. Ten of the 16 combinations of the IU can be utilized to prepare worksheets. Figure 5 shows 4 of these combinations used to represent single digit multiplication.

Figure 5

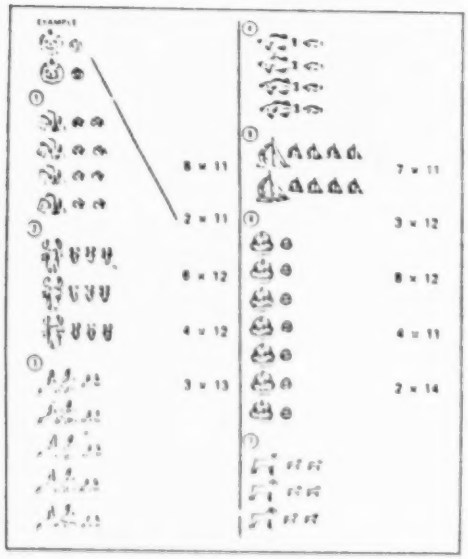
Item 1 Write/Write



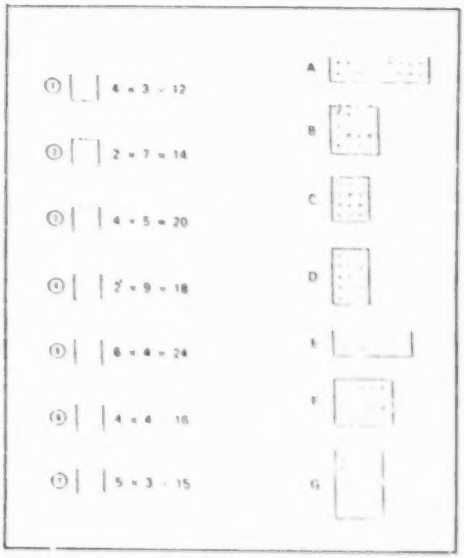
Item 2 Display/Write



Item 3 Display/Write



Item 4 Write/Identify



Examine item 3. It shows a worksheet developed form the Display/Write format. One could easily exchange the positions of the pictures and the numerals to produce an activity sheet with a symbolic (write) input and a pictorial (identify) response. Each of the pictures could be changed (e.g., pumpkins to sailboats) and a new worksheet created. Such an approach would enable the child to practice and to generalize.

Algorithms

The algorithms or approaches used in schools and in school texts were developed as we moved more into paper-pencil arithmetic. No one has inferred that these approaches are more meaningful than others. The paper-pencil algorithms originally became popular because they are efficient and because they take up comparatively little space on paper. Schools contributed to their popularity because they fit nicely into the textbook, they are easily prepared (e.g., dittos can be cranked out quickly) and they are easily manageable (i.e., they can be handed out to the entire class in a few seconds).

But, the sole reliance on the traditional textbook algorithm seriously limits our ability to develop meanings, to demonstrate alternative approaches, to conduct appraisals and to assist individual children whose performance is highly unusual. It is ironic, that in the days of high-tech and the ever increasing popularity of hand-held calculators that we insist on the right-hand column as the sole beginning point when the calculator works from left-to-right (Pelligrino & Goldman, 1987).

It is important, therefore, that the teacher become familiar with a variety of algorithms, and be able to detect the subtle differences between what seems to be a strange approach on the part of the child and the use of a different algorithm. Our position is that instruction will be more efficient and learning more effective when the teacher utilizes a least correction model. That is, the teacher should correct only the error that is occurring and not the overall performance style of the child. Examine the two items below.

653	653
+544	-544
1200	100
90	40
7	8
1297	108

Note that in each instance the child worked from left-to-right and that in each instance the answer is incorrect. Our experience is that the teacher is likely to begin by informing the child that going from left-to-right is wrong and that he/she should begin on the right. But, where did the actual errors occur? Clearly, not in the use of the alternative algorithm. The errors were computational and it is in computation that the correction should made. Once the child has mastered the left-to-right algorithm and the arithmetic combinations, the teacher might then suggest that there are other approaches, one being right-to-left.

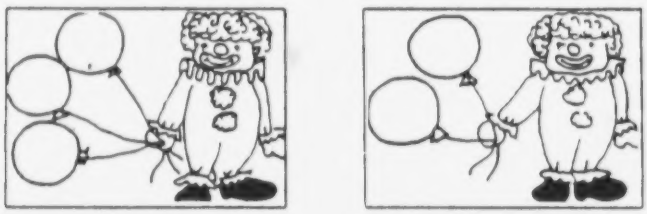
Approach Computation

In our introduction, we indicated that the primary reasons for computation are to provide the correct response to problems and to provide accurate data for dealing with real-life problems. The illustrations that follow (see Figure 6) are taken from a problem solving test that is presently under development (Cawley & Miller, in progress).

These illustrations focus on two conceptual schemes, clowns and animal trainers. The teacher can develop the needed background and vocabulary through language arts activities, visits to a circus or zoo or through films or other media. Developing the background is important for the more the children know about a topic, the less they will be burdened by extraneous concerns.

Figure 6

A.)



B.)

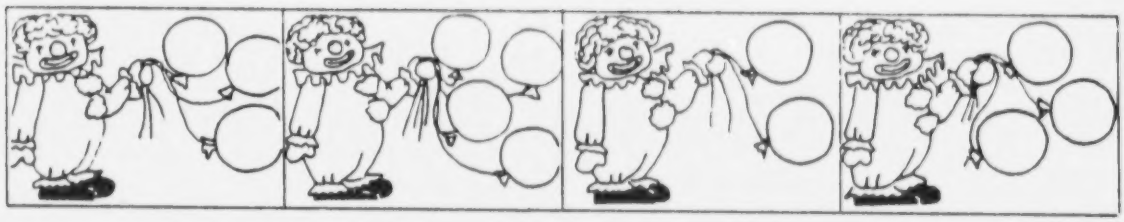


Figure 6A displays two pictures of clowns with balloons. The input is:

1. See the clowns.
2. Each clown has some balloons.
3. How many balloons do the clowns have in all?

The initial statement directs the attention of the child to the context. The second statement directs the attention of the child to the items that will be joined (i.e., added together). The third statement signals the joining action. Note carefully the use of the indefinite quantifier "some" in the second statement. Note also that no mention is made of the number of balloons each clown possesses. There are specific reasons for phrasing the statements in this manner. First, the use of the indefinite quantifier forces the child to utilize the pictures to identify the number of balloons possessed by each clown. Had the numbers been given, the child would not use the pictures. Second, the indefinite quantifier forces the child to count the objects and to utilize counting as the process for joining. Again, had the number been given the child would likely have searched 3 + 2 only to realize that he or she is unable to amalgamate them or does not remember these facts. Thus, with the indefinite quantifier the child can (must) meaningfully approach the process of addition without as yet having dealt with the so-called "facts." The same is true with Figure 6B, only this time the child is joining/adding 4 sets. The child is utilizing a form of the associative property of addition and it is now that the importance of our earlier concerns for "counting on" emerge. The teacher could vary the task to encourage counting on by first displaying one picture, then another and so forth.

We come now to the point where we must differentiate between the processes and understandings of addition, subtraction, multiplication and division as being different from the rote memorization and application of the traditional algorithm. To do so, it is important to group the operations and the proficiencies of the children with mental disabilities who perform them into selected stages. These stages are (a) single digit by single digit, (b) two digits by two digits and (c) three or more digits by combinations of one or more digits. Once they understand the operation, children can be helped to memorize the single digit combinations. But, memorization should not precede understanding.

Once they understand place value and renaming, children with mental disabilities can be introduced to operations with combinations of two digits. Proof of the levels of understandings should be determined via manipulative and pictorial representations and by their ability to perform two digit calculations with expanded notation. After they are able to perform two digit calculations at high rates of percent correct and at the fastest possible rates in time, the children can move to combinations with three or more digits by one or more digits.

Three digit by other combinations pose unique problems for the teacher and the child. In one of our companion studies (Magwili, in progress) it was found that only 15 percent of nonhandicapped children in the fifth grade and only 54 percent of nonhandicapped children in the eighth grade successfully answered:

$$46 \overline{)4060}$$

In another study among children with learning disabilities (Miller & Milam, in press) it was found that the majority of errors on problems of the above type were in subtraction and multiplication and not in division, per se.

Magwili (in progress) took the item shown above and broke it down into many of its component parts, Figure 7. Her data show that 85 percent of the fifth grade sample got the single digit item correct. Performance fell below the 50 percent level on the single digit divided into three digit example.

Figure 7

$4/\overline{4}$ $4/\overline{40}$ $4/\overline{38}$ $6/\overline{38}$ $4/\overline{4060}$ $6/\overline{40}$ $6/\overline{400}$

$4/\overline{380}$ $6/\overline{4060}$ $46/\overline{380}$ $6/\overline{380}$ $6/\overline{380}$ $46/\overline{406}$ $46/\overline{4060}$

One other recent study (Cawley, Miller & Watts, in progress) showed that only 30 percent of a sample of senior high school children with mental disabilities accurately performed division problems with two digit divisors. Clearly, the curriculum decision to emphasize "long division" via the traditional approach is questionable. An alternative is needed and that alternative exists as follows:

1. Develop a clear and precise understanding of each operation
2. Develop computational proficiency with two digit by two digit combinations through three digit by one digit combinations
3. Develop proficiency with estimation as a process and as a competency (Cawley, 1985)
4. Develop proficiency with the hand-held calculator
5. Minimize worksheet/ditto and paper-pencil activities
6. Make problem solving and situational computation the dominate school activity

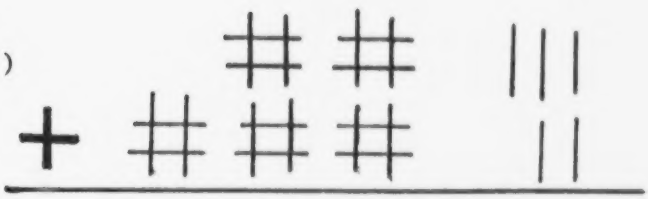
Expanded Notation: The Last Step Before Computation

Counting beyond 9 involves regrouping when objects are used and renaming when symbols are used. Exchanges between columns in items with two or more places involve the same combinations. Figure 8 shows both a manipulative and a symbolic representation of two digit by two digit addition. The exchange that takes place between formats generally begins when the child has erred in the symbolic format. In this situation, the teacher will gather a set of materials and represent the item for the child. The child may even build the representation. Our experience suggests that the direct exchange between the manipulative and the traditional symbolic representation must be mediated by an intermediate activity, namely expanded notation. In effect, expanded notation is the format through which place value is expressed.

Expanded notation involves the numerical representation of each place value position. Thus, 23 becomes 20 + 3 and 265 becomes 200 + 60 + 5. In our illustration in Figure 8, the manipulative representation of 23 plus 32 would be shown as 20 + 3 plus 30 + 2 (Figure 8A). The symbolic representation in expanded notation is shown in Figure 8B, and the traditional symbolic representation is shown in Figure 8C. The inverse would take place if we began with the symbolic form.

Figure 8

A.)



= group of 10 sticks

B.)

$$\begin{array}{r} 20 + 3 \\ +\ 30 + 2 \\ \hline \end{array}$$

C.)

$$\begin{array}{r} 23 \\ +\ 32 \\ \hline \end{array}$$

What this does is provide a symbolic display of the manipulative form thereby helping the child to see more directly the relationship between the expanded format that exists with manipulatives and the traditional format with closed columns. These three components, the manipulative representation, expanded notation and the traditional closed column form, enable us to perform two important functions. First, they enable us to give proper meaning and understanding to the operations and skills that are so important to the child with mild mental disabilities. Second, they enable us to teach the operations in any sequence so that we can meet needs independent of the influence of prerequisite deficits (e.g., we can teach multiplication to a child who does not subtract).

Computation

As stated at the start of this chapter, we do not intend to provide a step-by-step manual of how to teach computation to mildly handicapped children. What we will do here at the conclusion of this work is to provide an example of how our theories and methods could be used in a class for these children. As we noted earlier, we have found that the traditional order in which computation is taught (e.g. addition, then subtraction, followed by multiplication, and finally division) is not at all necessary for students to understand and perform computations. We have been bound by tradition, and how we were taught when we ourselves were youngsters. It should be up to us as modern teachers to be willing to accept changes in time honored methods if these changes are for the good of our students. What we propose here is the teaching of multiplication, independent of the other operations of addition and subtraction. The reader should bear in mind however, that we did note that the basic concepts of counting and place value (expanded notation) must be mastered before any instruction beyond single digit computation is attempted.

For our example we will begin instruction in multiplication with the fact "4 x 3 = 12". We are not going to have the child memorize this fact, but will have the child understand this fact and generalize from this fact to all other multiplication facts. We will make extensive use of arrays in our lessons, and would begin as follows:

1. Present the child an array of manipulatives (beans, blocks, etc.) as shown:

   xxx xxx xxx xxx

   Have the child count the number of things in each set and the number of sets (here we are using the concept of equal sets). Say: "Make one like mine." Child copies arrays and recounts.

2. Rearrange your materials into the array shown below:

   xxx
   xxx
   xxx
   xxx

   Say: "Make one like mine." Child copies array and notes that there are 4 rows of 3, the same number as before the array was produced.

3. Reform your array as shown:

xxxx
xxxx
xxxx

Say: "Make one like mine." Child copies array and notes that there are now 3 rows of 4, and that the array is now turned sideways. The child notes that there are the same number or items regardless of the way they are presented. You have now presented the commutative property of multiplication (e.g. 3 x 4 is the equivalent of 4 x 3). This is an important step in the teaching of multiplication as once the child understands the commutative property, the number of facts that must ultimately be learned is almost cut in half. Have the child change the array several times to cement the concept.

4. You could now present the various permutations of the first fact, 4 x 3. Arrange an array as shown:

xxx xxx
xxx
xxx

Say: "Make one like mine." Child copies array. Note that there are now two facts represented: 1 x 3 and 3 x 3. Stress the idea that there are still the same number of items in the array, but they are now in two sets. [You recognize this as the distributive property of multiplication.] Use "counting on" to demonstrate that 1 x 3 and 3 x 3 is the same as 4 x 3. You can divide the array into the other permutations, always demonstrating that no matter what formation is give, the items always represent 4 x 3.

Note: We recommend that the numerical representation of each expression be used starting at step four if not before. The numerical expression could be used throughout if desired, simply present the expression on a card which the child can view as he or she manipulates the items as instruction progresses. At all steps, the child could also supplement the manipulatives with paper and pencil representations of each array. The teacher simply says: "Draw it."

5. The next step is the presentation of multiplication "problems" rather than simple "facts". This occurs when numerals of 10 or more are used. For example, the problem:

$$\begin{array}{r} 13 \\ \times\ 3 \\ \hline \end{array}$$

would become cumbersome if 3 rows of 13 ones were required. We have already noted that knowledge of place value and expanded notation is required prior to instruction in computation, and it is at this point in the multiplication instruction when this knowledge is required. The child must know that 13 is made up of one 10 and 3 ones. The "+" sign could be used

here to indicate "and" rather than an operation, so the expression 10 + 3 = 13 does not presuppose any knowledge of addition on the part of the student.

It is good to use items such as popsicle sticks, etc. which can be bundled into groups of 10 for these problems. For the above problem, make an array as follows:

(# represents one bundle of 10)

#	xxx	10 + 3
#	xxx	10 + 3
#	xxx	10 + 3
		3 9

In order to solve this problem, the child counts the number of items in the ones column (9) and the number of bundles of 10 (3) and arrives at the correct answer (39).

6. Ah ha! You are now thinking, "What happens when there are more than 9 tens in the ones column?" Remember that the student is proficient in expanded notation and place value. When presented with the problem:

$$\begin{array}{r} 13 \\ \times\ 4 \end{array}$$

the student would form an array as shown.

#	xxx	10 + 3
#	xxx	10 + 3
#	xxx	10 + 3
#	xxx	10 + 3

The child counts the number of items in the ones column and when 10 is reached, he or she automatically "bundles" the ten, places it with the other bundles of ten, and continues counting the remaining ones (2). The student then counts the number of bundles in the 10's columns and notes that there are 5, thus arriving at the correct answer of 52.

#		10
#	xx	10 + 2
#		10
#		10
#		10
		5 2

7. The teacher could continue to provide manipulatives indefinitely as the child works multiplication problems. However, the student <u>understands</u> <u>the concept</u> of multiplication and is able to perform the operation which is what the instruction is all about in the first place. At this point, we recommend that the teacher provide the students with a hand-held calculator.

The above example demonstrates clearly how the operations used in mathematics could be taught independent of each other. Basically the same steps could be used for either of the other three operations in mathematics.

SUMMARY

All too frequently, we find ourselves caught in the web spun by our predecessors in teaching only to find that we are reluctant to change our views and methods. It was our aim in this work to offer alternatives to traditional methods and to alert practitioners to the dangers of stagnation in mathematics teaching.

We must be aware that students are able to learn mathematics in alternative ways. We must also be able to accept these alternatives when offered. It is our hope that this work will provide a thought provoking stimulus for action on the part of those who teach mathematics to the mildly handicapped.

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